SELECTION METHOD OF GRINDING MACHINE AND AIR CLASSIFIER IN GRINDING-CLASSIFICATION PROCESS BY USING FSFDMW-TOPSIS

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ABSTRACT: Type selection of grinding machines and air classifiers is a critical issue in dry grinding–classification process design, particularly under uncertain environments where statistical data are unavailable and expert judgments dominate decision making. This study proposes a fuzzy group decision-making framework integrating fuzzy equivalence clustering, fuzzy score function with decision makers' weights (FSFDMW), and TOPSIS to enhance selection reliability. First, main criteria are identified using fuzzy equivalence clustering. Then, an n-dimensional fuzzy environment is constructed to determine the weights of decision makers and criteria. Finally, a TOPSIS procedure based on fuzzy score functions is applied to rank alternatives. Application to the dental gypsum grinding–classification process shows that the impact mill achieves the highest priority value (0.742), while the MS type air classifier obtains the highest priority value (0.96417). The proposed framework improves decision accuracy while maintaining computational simplicity.

KEY WORDS: Decision Makers` Weight; fuzzy score function; FSFDMW-TOPSIS method; grinding machine; air classifier; grinding-classification process

1. INTRODUCTION

In dry grinding-classification process, the type selection of grinding machine and air classifier is one of the most important issues in process design. The optimal process design that meets the specifications of the grinding-classification process can be achieved by selecting the most suitable types of grinding machines and air classifiers among the available alternatives.

In order to select the appropriate grinding machine and air classifier types required in the process, first, the set of selected proposals and the criteria affecting selection must be determined, and the weight of the importance of those criteria must be calculated. Moreover, there must be a method to make a reasonable choice. However, there exists indefinite ambiguity regarding how to select the criteria influencing the reliable type selection of grinding machine and air classifier in grinding-classification process design, how to determine the important criteria, and how to perform a reliable selection. In addition, no statistical data can be provided to resolve these issues. Therefore, this problem must be solved by using the FSFDMW method based on the subjective opinion data of technicians and experts in this field.

The fuzzy set decision making problem aims to select the optimal proposal by integrating the expert opinions of decision makers, given the set of selected proposals, the set of different criteria to be considered, and the set of decision makers under uncertainty in the decision-making environment.

One of the typical techniques for determining the most reasonable selection is the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). In the classical TOPSIS method, the decision-making environment has no uncertainty [1]. To solve decision-making problems under uncertainty, fuzzy TOPSIS methods have been proposed.

Chen [2] proposed an extension of TOPSIS for group decision making under a fuzzy environment. Jahanshahloo et al. [3] extended the TOPSIS concept for solving multi-criteria decision-making problems with fuzzy data. Applications of fuzzy group decision making using fuzzy TOPSIS have been extensively studied. Chang [4] developed a fuzzy TOPSIS method for optimal initial training aircraft evaluation. Chu [5] proposed a fuzzy TOPSIS model for aircraft trajectory location selection. Gligoric et al. [6] developed a model for arrow location selection in complex ore-body rocks using fuzzy TOPSIS combined with network optimization. Doukas [7] conducted a study on renewable energy source selection using fuzzy TOPSIS associated with carbon-gas emission reduction.

Fuzzy Hierarchical Analysis (AHP) is also widely used in group decision making problems. Ghodsipour [8] reported that fuzzy hierarchy analysis is one of the most comprehensive systems for multi-criteria decision making. The classical fuzzy hierarchy analysis method performs consistency verification after pairwise comparison questionnaires and constructs a decision matrix through pairwise comparisons.

Safari [9] used fuzzy hierarchical analysis (AHP) to select mineral processing equipment considering eight criteria in iron ore mines. Ataei [10] applied the Monte Carlo hierarchical analysis method to select the most suitable underground mining method at Jashear bauxite mine. Rahimdel [11] used fuzzy hierarchical analysis to select the most suitable crusher among major available crushers. Karimnia [12] determined the best mining method using fuzzy hierarchical analysis in a salt mine in Iran.

In set decision making problems, studies combining fuzzy hierarchical analysis (AHP) and fuzzy TOPSIS have been proposed, in which the weights of criteria are determined by fuzzy AHP, and the rational selection is performed by fuzzy TOPSIS [13], [14].

Many researchers have introduced intuitionistic fuzzy sets (IFS) with TOPSIS to provide hybrid approaches for multi-criteria decision-making problems [15], [16]. Other studies extended fuzzy TOPSIS for multi-criteria decision making on interval-valued intuitionistic fuzzy information and uncertain fuzzy information [18]–[21]. Recently, methods have been proposed to solve uncertain multi-criteria problems in uncertain fuzzy environments. Xu and Zhang [19] proposed a fuzzy multi-attribute decision making method by TOPSIS with incomplete fuzzy information. Liu et al. [22] proposed a TOPSIS method for uncertain multi-criteria decision-making problems.

However, the TOPSIS method, AHP method, and their combination methods discussed above are limited either to a single decision maker or do not consider the weight of the decision makers' evaluation levels in group decision making. The accuracy of fuzzy group decision making is strongly related to the limitations, evaluation levels, and expertise of decision makers, as well as to the number of decision makers.

Yue [17] proposed an extension of TOPSIS to determine the weights of decision makers in group decision making problems with uncertain information. Zhang [23] developed two nonlinear optimization models for various criteria group decision making problems with uncertain fuzzy information, deriving explicit formulas for the weights of decision makers and criteria. However, the drawbacks of the method in [23] are that, in various criteria group decision making, the weights of decision makers differ due to internal considerations and value personalities according to each criterion, while the weights of criteria are not considered to depend on the number of decision makers, increasing computational complexity in practical applications.

In previous studies on determining rational alternatives, the criteria affecting the alternatives are assumed to be already known, and no study has proposed a method for selecting these criteria.

To address this issue, in this paper, we first propose a method to select the main criteria influencing the selection by means of fuzzy equivalence clustering using a fuzzy score function with the weights of decision makers. Next, we define an n-dimensional fuzzy environment with respect to the number of decision makers and develop a method to calculate the weights of decision makers for each criterion and for the whole set of criteria using simple averaging.

Then, we develop a method to calculate the weights of criteria using a scoring function of each criterion with the decision makers' weights for each criterion, as well as a scoring function of the criteria with the decision makers' weights for the whole set of criteria. Finally, we propose a TOPSIS decision-making method using a fuzzy score function with the weights of decision makers and apply it to select the appropriate type of grinding machine and air classifier for dental gypsum grinding.

The main advantage of this method over currently available methods is that it improves the accuracy of group decision making by calculating the weights of decision makers more realistically, while maintaining computational simplicity through simple averaging operations.

2. METHODOLOGY

2.1. Research Plan and Method

In this section, the research plan and method are explained. The first step begins with conducting a literature study using various references.

In order to make a reasonable selection among the selection proposals, several criteria influencing the selection must first be determined. Among the possible criteria affecting group decision making for rational selection, the basic and main criteria can be selected using multivariate analysis based on statistical data. However, this method cannot be applied for qualitative criteria where statistical data cannot be obtained.

Therefore, we propose a method to select the main criteria among the possible criteria to consider based on the knowledge of decision makers (technicians and experts).

Let the set of possible criteria affecting rational selection be denoted as $C = \{c_1, c_2, \dots, c_m\}$, and the set of decision makers be denoted as $Z = \{Z_1, Z_2, \dots, Z_n\}$.

The subjective opinion data of decision makers for selecting the main criteria is given in Table 1.

In Table 1, $e_{kj} \in [0,1]$, where the k-th decision maker denotes his subjective opinion as the importance degree of the j-th criterion. The value 0 represents a secondary criterion, the value 1 represents a main criterion, and a value between [0,1] represents its relative importance.

Table 1: The subjective opinion data of decision makers for selecting the main criteria

C	c_1	c_2	…	c_j	…	c_m
Z_1	e_{11}	e_{12}	…	e_{1j}	…	e_{1m}
Z_2	e_{21}	e_{22}	…	e_{2j}	…	e_{2m}
⋮	⋮	⋮	…	⋮	…	⋮
Z_k	e_{k1}	e_{k2}	…	e_{kj}	…	e_{km}
⋮	⋮	⋮	…	⋮	…	⋮
Z_n	e_{n1}	e_{n2}	…	e_{nj}	…	e_{nm}

2.2. Main Criteria Selection by Fuzzy Equivalence Clustering

Definition 1: The k-th decision maker's weight for the j-th criterion is defined as:

$$w'_{kj} = 1 - \left| \left(\frac{1}{n}\sum_{l=1}^{n} e_{lj}\right) - e_{lj} \right|, \quad (k = 1, 2, \cdots, n, \ \ j = 1, 2, \cdots, m) \tag{1}$$

After standardization:

$$w_{kj} = \frac{w'_{kj}}{\sum_{l=1}^{n} w'_{lj}}, \ w_{kj} \geq 0 \ , \ \sum_{k=1}^{n} w_{kj} = 1 \ (j = 1, 2, \cdots, m) \tag{2}$$

Definition 2: The fuzzy score function values with decision maker's weight for the j-th criterion are defined as:

$$S(x_j) = \frac{1}{n}\sum_{k=1}^{n} w_{kj} e_{kj} \ , (j = 1, 2, \cdots, m) \tag{3}$$

The algorithm for selecting the main criteria is structured as follows:

Step 1: Construct the subjective opinion data of decision makers.

Step 2: Calculate the decision makers' weights using Eqs. (1) and (2).

Step 3: Calculate the fuzzy score function values using Eq. (3).

Step 4: Define the two-dimensional fuzzy equivalence relation:

$$\underset{\sim}{R}_{C}(x_\upsilon, x_\omega) = \begin{cases} 1 & , \ \upsilon = \omega \\ \min\{\bar{e}_\upsilon, \bar{e}_\omega\} & , \ \upsilon \neq \omega \end{cases} \quad \left(\upsilon, \omega = \overline{1, m}\right) \tag{4}$$

where

$$\bar{e}_u = S(x_u) = \frac{1}{n}\sum_{k=1}^{n} w_{ku} e_{ku} \tag{5}$$

represents the fuzzy score function value with decision maker's weight.

Step 5: Define the fuzzy equivalence classification level α.

Step 6: Determine the α-cut matrix:

$$R_{C\lambda}\left(x_\upsilon, x_\omega\right) = \begin{cases} 1 \quad , \quad \underset{\sim}{R}_{C}\left(x_\upsilon, x_\omega\right) \geq \lambda \\ 0 \quad , \quad \underset{\sim}{R}_{C}\left(x_\upsilon, x_\omega\right) < \lambda \end{cases} \tag{6}$$

Step 7: Calculate the maximum fuzzy score function value in the fuzzy equivalence relation matrix:

$$\bar{e}_\eta = \max_{1 \leq \tau \leq m} \{\bar{e}_\tau\} \tag{7}$$

Step 8: Determine fuzzy equivalence classification using the α-cut matrix.

If $r_{ij} \geq \alpha$, criterion $c_i$and criterion $c_j$belong to the same fuzzy equivalence class.

The criteria belonging to the fuzzy equivalence class, such as the η-th criterion obtained by Eq. (7), are selected as the main criteria.

The α-cut matrix is defined to reflect practical requirements.

2.3. Main Criteria Weight Determination by Using FSFDMW

In this section, we propose a fuzzy group decision making method to determine the weights of various criteria when the fuzzy environment is given as an n-dimensional fuzzy environment determined by n decision makers.

Because of different experience and characteristics of decision makers, the weights of decision makers for each criterion are different. Therefore, the weight for the set of whole criteria is related to the weight of decision makers for each criterion.

We assume: (i) The importance degree of criteria varies according to the selection; (ii) The evaluation level degree of decision makers on the whole criteria depends on their evaluation on each criterion; and (iii) The number of weights of criteria depends on the number of decision makers.

Let H be the set of n-dimensional fuzzy elements.

Definition 3: Given a map from criteria set $C$to a subset of n-dimensional values in [0,1], this map determines an n-dimensional fuzzy set (n-DFS). Its membership function is expressed as:

$$E_n = (< c_j, h_{E_n}(c_j) > | c_j \in C,\ j = 1,2,\cdots,m) \tag{8}$$

where the value denotes the degree of probability that an element belongs to the n-dimensional fuzzy set.

The k-th decision maker's weight for an element of n-DFS for j-th criterion is defined as:.

$$w_{jk} = \frac{w'_{jk}}{\sum_{p=1}^{n} w'_{jp}},\ (k = 1,2,\cdots,n,\ j = 1,2,\cdots,m) \tag{9}$$

If evaluation level degrees are equal, then the weight becomes uniform.

Definition 4: The fuzzy score function with weights for j-th criterion is defined as:

$$S(h_j) = \frac{1}{n}\sum_{k=1}^{n} w_{j_k} h_{jk},\ (j = 1,2,\cdots,m) \tag{10}$$

If evaluation level degrees are equal, the function simplifies accordingly.

The weights of evaluation level degrees of decision makers are defined as:

$$w'_k = 1 - \frac{1}{m}\left[\sum_{j=1}^{m} \left| S(h_j) - h_{jk} \right|\right], (k = 1,2,\cdots,n) \tag{11}$$

Definition 5: The fuzzy score function with weights for criteria is defined as:

$$S_j(H_n) = \frac{1}{n}\sum_{k=1}^{n} w_k h_{jk},\ (j = 1,2,\cdots,m) \tag{12}$$

The importance degree weight for each criterion is determined as:

$$W_{c_j} = \frac{S_j(H_n)}{\sum_{l=1}^{m} S_l(H_n)},\ (j = 1,2,\cdots,m),\ W_{c_j} \geq 0\ \sum_{j=1}^{m} W_{c_j} = 1 \tag{13}$$

2.4. TOPSIS Method by Using FSFDMW

In this section, we propose the TOPSIS method by using the fuzzy score function with decision maker's weight.

Let:

- V be the set of selections
- C be the set of criteria
- Z be the set of decision makers

Dominance evaluation opinion data is given in Table 2.

Table 2: Dominance evaluation opinion data of n decision makers for j-th criterion c_j on T selections

V	V_1	V_2	$\cdots$	V_t	$\cdots$	V_T
Z_1	h_{1j1}	h_{2j1}	$\cdots$	h_{tj1}	$\cdots$	h_{Tj1}
Z_2	h_{1j2}	h_{2j2}	$\cdots$	h_{tj2}	$\cdots$	h_{Tj2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Z_k	h_{1jk}	h_{2jk}	$\cdots$	h_{tjk}	$\cdots$	h_{Tjk}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Z_n	h_{1jn}	h_{2jn}	$\cdots$	h_{tjn}	$\cdots$	h_{Tjn}

Definition 6: The k-th decision maker's weight for n-DFS elements of t-th selection on j-th criterion is defined as:

$$S(h_{tj}) = \frac{1}{n}\sum_{k=1}^{n} w_{tjk} h_{tjk} \ , (t = 1, 2, \cdots, T) \tag{14}$$

The fuzzy score function with weights for t-th selection is:

$$w'_{jk} = 1 - \frac{1}{T}\left[\sum_{t=1}^{T}\left| S(h_{tj}) - h_{tjk}\right|\right], (k = 1, 2, \cdots, n) \tag{15}$$

Weights of evaluation level degrees for selections are defined as:

$$w_{jk} = \frac{w'_{jk}}{\sum_{p=1}^{n} w'_{jp}} , (k = 1, 2, \cdots, n) \tag{16}$$

Definition 7: The fuzzy score function with weights for selections is defined as:

$$S'_{tj}(H_{nj}) = \frac{1}{n}\sum_{k=1}^{n} w_{tk} h_{tjk} \ ,(t=1,2,\cdots,T) \tag{17}$$

After normalization:

$$S_{tj}(H_{nj}) = \frac{S'_{tj}(H_{nj})}{\sum_{t=1}^{T} S'_{tj}(H_{nj})} \ ,(j=1,2,\cdots,m),\ \ S_{tj}(H_{nj}) \geq 0,\ \sum_{t=1}^{T} S_{tj}(H_{nj}) = 1 \tag{18}$$

The fuzzy score function matrix is shown in Table 3.

Table 3: Fuzzy score function value matrix with weights of decision makers

V	c_1	c_2	…	c_j	…	c_m
V_1	$S_{11}(H_{n1})$	$S_{12}(H_{n2})$	…	$S_{1j}(H_{nj})$	…	$S_{1m}(H_{nm})$
V_2	$S_{21}(H_{n1})$	$S_{22}(H_{n2})$	…	$S_{2j}(H_{nj})$	…	$S_{2m}(H_{nm})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
V_t	$S_{t1}(H_{n1})$	$S_{t2}(H_{n2})$	…	$S_{tj}(H_{nj})$	…	$S_{tm}(H_{nm})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
V_T	$S_{T1}(H_{n1})$	$S_{T2}(H_{n2})$	…	$S_{Tj}(H_{nj})$	…	$S_{Tm}(H_{nm})$

TOPSIS Algorithm with FSFDMW:

Step 1: Construct dominance evaluation opinion data for each criterion.

Step 2: Calculate and normalize decision maker weights using Eq. (14).

Step 3: Calculate fuzzy score functions using Eq. (15).

Step 4: Calculate and normalize evaluation level weights using Eq. (16).

Step 5: Compute fuzzy score function values using Eqs. (17) and (18).

Step 6: Construct the decision making matrix:

$$y_{tj} = W_{c_j} S_{tj}(H_{nj})(t=1,2,\cdots,T,\ j=1,2,\cdots,m) \tag{19}$$

Table 4: Decision making matrix

	c_1	c_2	…	c_j	…	c_m
V_1	y_{11}	y_{12}	…	y_{1j}	…	y_{1m}
V_2	y_{21}	y_{22}	…	y_{2j}	…	y_{2m}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
V_t	y_{t1}	y_{t2}	…	y_{tj}	…	y_{tm}
⋮	⋮	⋮	⋮	⋮	⋮	⋮
V_T	y_{T1}	y_{T2}	…	y_{Tj}	…	y_{Tm}

Step 7: Determine Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS):

$$\begin{cases} A^{+} = (y_1^{+}, y_2^{+}, \cdots y_m^{+}) \\ A^{-} = (y_1^{-}, y_2^{-}, \cdots y_m^{-}) \end{cases} \tag{20}$$

$$\begin{cases} y_j^{+} = \{(\max\limits_{1 \le t \le T} y_{tj} \mid j \in J_1), (\min\limits_{1 \le t \le T} y_{tj} \mid j \in J_2)\} \quad (j = 1, 2, \cdots, m) \\ y_j^{-} = \{(\min\limits_{1 \le t \le T} y_{tj} \mid j \in J_1), (\max\limits_{1 \le t \le T} y_{tj} \mid j \in J_2)\} \quad (j = 1, 2, \cdots, m) \end{cases} \tag{21}$$

Step 8: Compute distances to PIS and NIS:

$$D_t^{+} = \sqrt{\sum_{j=1}^{m} \left(y_{tj} - y_j^{+}\right)^2} \quad (t = 1, 2, \cdots, T) \tag{22}$$

$$D_t^{-} = \sqrt{\sum_{j=1}^{m} \left(y_{tj} - y_j^{-}\right)^2} \quad (t = 1, 2, \cdots, T) \tag{23}$$

Step 9: Calculate priority value:

$$B_t = \frac{D_t^{-}}{D_t^{+} + D_t^{-}} \quad (t = 1, 2, \cdots, T) \tag{24}$$

Step 10: Rank selections.

The best selection is the one with the smallest distance to PIS and the largest distance to NIS.

3. TYPE SELECTION OF GRINDING MACHINE AND AIR CLASSIFIER IN DENTAL GYPSUM GRINDING-CLASSIFICATION PROCESS

3.1. Type Selection of Grinding Machine

With the rapid development of science and technology, the demand for new materials is increasing, particularly for powder materials requiring strict particle size distribution, specific particle shape, and extremely low impurity inclusion rates. To satisfy these requirements, grinding machines based on different grinding principles have been widely developed and applied in industry, and their structure and performance continue to be improved.

Since various types of grinding machines exhibit different structural characteristics and performance levels, the technical and economic efficiency of a grinding–classification process depends significantly on the selected mill type. Therefore, selecting the grinding machine most suitable for practical operating conditions is an essential requirement for improving the overall efficiency of the dental gypsum grinding–classification process.

In this study, the appropriate type-selection method for establishing the dental gypsum grinding–classification process is carried out based on the previously introduced fuzzy group decision-making methodology.

Currently, eight types of grinding machines are widely used in dry grinding processes:

- Tumbling ball mill (A)
- Centrifugal roller mill (B)
- Vibrating mill (C)
- Air-flow mill (D)
- Planetary mill (E)
- Stirred mill (F)
- Impact mill (G)
- High-pressure roller mill (H)

A suitable mill type is selected from these eight alternatives using the FSFDMW–TOPSIS method.

3.2.1. Setting the Main Criteria for Selecting Type of Grinding Mill

Ten technicians and experts are constructed as decision makers. The possible criteria considered for selecting a suitable mill type include:

- Fineness of grinding product
- Content of impurities
- Capacity of the mill
- Technical efficiency
- Specific power consumption
- Reliability
- Manufacturing possibility
- Manufacturing cost
- Continuity of grinding
- Ease of repair
- Suitability of operation
- Life of the mill

The opinion data of the decision makers are listed in Table 5.

Table 5: Opinion data of decision makers for selecting the main criteria

Fineness	Content of impurities	Capacit y	Technical Efficiency	Specific pow er consumpti on	Reliabili ty	Manufacturi ng Possibilit y	Manuf acturin g cost	Continuity ofworking	easy of rep air	suitability of ope ration	Life of m ill

1	0.9	0.9	0.7	0.3	0.5	0.7	0.3	0.3	0.3	0.4	0.4	0.4
2	0.9	0.8	0.9	0.4	0.5	0.8	0.4	0.4	0.4	0.4	0.4	0.5
3	0.8	0.7	0.9	0.3	0.7	0.8	0.4	0.4	0.3	0.4	0.3	0.4
4	0.9	0.7	0.9	0.4	0.7	0.7	0.3	0.3	0.3	0.4	0.4	0.4
5	0.7	0.7	0.7	0.4	0.7	0.7	0.4	0.3	0.3	0.3	0.5	0.5
6	0.7	0.6	0.7	0.4	0.6	0.7	0.5	0.4	0.3	0.4	0.3	0.4
7	0.8	0.7	0.8	0.5	0.7	0.8	0.4	0.3	0.3	0.5	0.4	0.5
8	0.8	0.6	0.8	0.4	0.7	0.9	0.4	0.2	0.2	0.3	0.3	0.4
9	0.9	0.7	0.8	0.3	0.6	0.9	0.5	0.3	0.2	0.3	0.4	0.5
10	0.9	0.8	0.8	0.3	0.7	0.9	0.5	0.2	0.3	0.3	0.4	0.4

First, the weights of the decision makers are calculated using (1) and (2). Then, the fuzzy score function values with the weights of decision makers are determined using (3).

Subsequently, the binary fuzzy equivalence relation $R$is calculated using (4) and (5), and represented in Table 6.

Table 6: Binary fuzzy equivalence relation R_c

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.72	0.80	0.37	0.64	0.79	0.41	0.31	0.29	0.37	0.38	0.44
2	0.72	1	0.72	0.37	0.64	0.72	0.41	0.31	0.29	0.37	0.38	0.44
3	0.80	0.72	1	0.37	0.64	0.79	0.41	0.31	0.29	0.37	0.38	0.44
4	0.37	0.37	0.37	1	0.37	0.37	0.37	0.31	0.29	0.37	0.37	0.37
5	0.64	0.64	0.64	0.37	1	0.64	0.41	0.31	0.29	0.37	0.38	0.44
6	0.79	0.72	0.79	0.37	0.64	1	0.41	0.31	0.29	0.37	0.38	0.44
7	0.41	0.41	0.41	0.37	0.41	0.41	1	0.31	0.29	0.37	0.38	0.41
8	0.31	0.31	0.31	0.31	0.31	0.31	0.31	1	0.29	0.31	0.31	0.31
9	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	1	0.29	0.29	0.29
10	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.31	0.29	1	0.37	0.37
11	0.38	0.38	0.38	0.37	0.38	0.38	0.38	0.31	0.29	0.37	1	0.38
12	0.44	0.44	0.44	0.37	0.44	0.44	0.41	0.31	0.29	0.37	0.38	1

The α-cut matrix is calculated using (6), and after applying (7), the fuzzy equivalence classification is obtained.

According to the agreement of technicians and experts, $\alpha = 0.64$ is selected. Based on this threshold, five criteria with decisive influence are identified as the main appraisal criteria:

1. Fineness of grinding product
2. Content of impurities
3. Capacity of the mill
4. Specific power consumption
5. Reliability of the mill

3.1.2. Determination of the Weights of the Main Criteria Using FSFDMW

The importance weights of the five selected criteria are determined using the subjective opinion data of the ten decision makers (Table 7).

Table 7: Opinion data of decision makers for determining the weight value of the main criteria

	Fitness of grinding product	Content of impurities	Capacity of mill	Specific power consumption	reliability of mill
1	0.8	0.8	0.5	0.5	1
2	1	0.5	0.4	0.2	0.7
3	0.7	0.6	0.5	0.3	1
4	1	0.7	0.5	0.4	0.8
5	0.8	0.6	0.6	0.5	0.6
6	0.7	0.7	0.7	0.5	0.6
7	0.6	0.6	0.8	0.7	0.8
8	0.8	0.6	0.7	0.7	0.8
9	0.7	0.6	0.5	0.3	0.6
10	0.8	0.6	0.5	0.4	0.8

The normalized weight values of decision makers for each criterion are calculated using (9), as shown in Table 8.

Table 8: Normalized weight value of the decision makers for each criterion

	Fitness of grinding product, x_1	Content of impurities, x_2	Capacity of mill, x_3	Specific power consumption, x_4	Reliability of mill, x_5
W_{j1}	0.0699665	0.0699665	0.0599713	0.0499761	0.0499761
W_{j2}	0.0600789	0.0600789	0.0700921	0.0600789	0.0500658
W_{j3}	0.0699671	0.0699671	0.0799624	0.0499765	0.0599718
W_{j4}	0.0701910	0.0601637	0.0701910	0.0601637	0.0501365
W_{j5}	0.0798269	0.0498918	0.0698485	0.0498918	0.0598702
W_{j6}	0.0699798	0.0699798	0.0599827	0.0699798	0.0499855
W_{j7}	0.0600214	0.0600214	0.0700250	0.0700250	0.0600214
W_{j8}	0.0798921	0.0699056	0.0599191	0.0499326	0.0499326
W_{j9}	0.0699649	0.0499749	0.0699649	0.0599699	0.0599699
W_{j10}	0.0600509	0.0600510	0.0700595	0.0600510	0.0500425

Then, using (10)–(13), the fuzzy score function values and normalized weight values of the criteria are obtained (Table 9).

From Table 9, the weight ranking of the criteria is:

1. Fineness of grinding product
2. Reliability of mill
3. Content of impurities
4. Capacity of mill
5. Specific power consumption

This ranking is used in the subsequent TOPSIS evaluation.

3.1.3. Selection of Grinding Mill Type Using FSFDMW–TOPSIS

The dominance evaluation opinion data of the ten decision makers for each criterion are given in Tables 10–14.

Table 10: Opinion data of decision makers for estimating dominance to the fitness of grinding product (c_1)

	A	B	C	D	E	F	G	H
1	0.7	0.6	0.8	0.9	0.9	0.8	0.6	0.5
2	0.8	0.7	0.8	0.9	0.8	0.7	0.7	0.6
3	0.7	0.7	0.7	0.9	0.9	0.7	0.8	0.5
4	0.8	0.6	0.7	0.8	0.9	0.7	0.7	0.4
5	0.7	0.7	0.8	0.8	0.9	0.8	0.7	0.4
6	0.5	0.6	0.8	0.8	0.9	0.8	0.7	0.5
7	0.8	0.5	0.9	0.9	0.9	0.7	0.6	0.4
8	0.8	0.5	0.7	0.8	0.9	0.7	0.6	0.4
9	0.7	0.6	0.8	0.7	0.9	0.7	0.7	0.4
10	0.7	0.6	0.8	0.9	0.9	0.8	0.6	0.5

Table 11: Opinion data of decision makers for estimating dominance to content of impurities (c_2)

	A	B	C	D	E	F	G	H
1	0.3	0.7	0.4	0.6	0.3	0.3	0.7	0.8
2	0.4	0.8	0.5	0.7	0.3	0.3	0.6	0.9
3	0.3	0.7	0.5	0.5	0.4	0.4	0.7	0.8
4	0.2	0.7	0.6	0.4	0.3	0.3	0.7	0.9
5	0.3	0.8	0.4	0.5	0.3	0.2	0.6	0.9
6	0.3	0.6	0.3	0.6	0.4	0.2	0.5	0.8
7	0.4	0.7	0.2	0.7	0.2	0.3	0.7	0.8
8	0.3	0.7	0.3	0.5	0.2	0.3	0.8	0.8
9	0.4	0.8	0.4	0.5	0.3	0.4	0.8	0.7
10	0.3	0.7	0.3	0.6	0.2	0.3	0.7	0.8

Table 12: Opinion data of decision makers for estimating dominance to capacity of mill (c_3)

	A	B	C	D	E	F	G	H
1	0.9	0.9	0.8	0.7	0.6	0.7	0.9	0.6
2	0.9	0.8	0.9	0.5	0.7	0.8	0.8	0.7
3	0.8	0.8	0.8	0.5	0.6	0.6	0.8	0.7
4	0.9	0.8	0.7	0.6	0.5	0.7	0.9	0.6
5	0.9	0.7	0.8	0.5	0.6	0.7	0.8	0.7
6	0.8	0.7	0.8	0.5	0.6	0.8	0.9	0.7
7	0.7	0.7	0.7	0.6	0.5	0.7	0.8	0.6
8	0.8	0.7	0.9	0.6	0.6	0.6	0.7	0.7
9	0.8	0.8	0.8	0.5	0.6	0.7	0.8	0.8
10	0.9	0.6	0.8	0.7	0.6	0.7	0.9	0.6

Table 13: Opinion data of decision makers for estimating dominance to specific power consumption(c_4)

	A	B	C	D	E	F	G	H
1	0.4	0.5	0.8	0.7	0.6	0.7	0.9	0.8
2	0.3	0.6	0.7	0.6	0.5	0.6	0.8	0.7
3	0.3	0.6	0.8	0.7	0.6	0.6	0.8	0.6
4	0.3	0.7	0.7	0.7	0.6	0.5	0.8	0.7
5	0.2	0.7	0.6	0.6	0.5	0.7	0.8	0.6
6	0.3	0.6	0.7	0.5	0.4	0.6	0.9	0.7
7	0.3	0.6	0.8	0.5	0.3	0.7	0.8	0.6
8	0.3	0.7	0.7	0.7	0.2	0.7	0.8	0.7
9	0.4	0.5	0.7	0.5	0.2	0.6	0.9	0.6
10	0.2	0.7	0.7	0.7	0.3	0.6	0.8	0.7

Table 14. Opinion data of decision makers for estimating dominance to reliability of mill (c_5)

	A	B	C	D	E	F	G	H
1	0.9	0.7	0.9	0.8	0.6	0.8	0.9	0.8
2	0.8	0.7	0.8	0.8	0.5	0.7	0.8	0.7
3	0.7	0.7	0.8	0.7	0.6	0.8	0.8	0.8
4	0.8	0.8	0.8	0.7	0.6	0.8	0.8	0.8
5	0.9	0.8	0.9	0.8	0.7	0.8	0.9	0.7
6	0.9	0.7	0.9	0.8	0.7	0.9	0.9	0.8
7	0.8	0.8	0.9	0.8	0.7	0.8	0.8	0.7
8	0.9	0.8	0.9	0.8	0.6	0.8	0.9	0.8
9	0.9	0.7	0.9	0.8	0.7	0.9	0.9	0.7
10	0.9	0.7	0.9	0.8	0.6	0.8	0.8	0.8

The dominance degree is defined within [0,1], where:

- 1 indicates the most dominant alternative
- 0 indicates no dominance

Using (14)–(18), the fuzzy score function value matrix with weights of decision makers is calculated (Table 15).

Table 15: Fuzzy score function value matrix with weights of decision makers

	Fitness of grinding product	Content of impurities	Capacity of mill	Specific power consumption	Reliability of mill
A	0.125061083	0.078391353	0.142283713	0.061799721	0.143884455
B	0.1070528	0.1765506	0.1267748	0.1279762	0.1250365
C	0.1356339	0.0950398	0.1356062	0.1481481	0.1471201
D	0.1459014	0.1370148	0.0969313	0.1276909	0.1317416
E	0.1572072	0.0707339	0.0998297	0.0850646	0.0650233
F	0.1305613	0.0736832	0.1185306	0.1297366	0.1370625
G	0.1197441	0.1676446	0.1407171	0.1710791	0.1420329
H	0.0788382	0.2009416	0.1393266	0.1485048	0.1080986

Then, the weighted decision-making matrix is obtained using (19) (Table 16).

Table 16: Decision making matrix with the fuzzy score function values and the weights of criteria

	Fitness of grinding product	Content of impurities	Capacity of mill	Specific power consumption	Reliability of mill
A	0.031260662	0.015429177	0.02519052	0.008431992	0.034493347
B	0.0267593	0.0347491	0.0224448	0.0174612	0.0299749
C	0.0339035	0.018706	0.0240083	0.0202134	0.035269
D	0.03647	0.0269676	0.0171611	0.0174222	0.0315823
E	0.039296	0.013922	0.0176743	0.0116063	0.015588
F	0.0326355	0.0145025	0.0209852	0.0177013	0.0328579
G	0.0299316	0.0329962	0.0249132	0.0233421	0.0340495
H	0.0197066	0.0395498	0.024667	0.0202621	0.0259144

The positive ideal solution (PIS) and negative ideal solution (NIS) are determined using (20) and (21). The distances to PIS and NIS are calculated using (22) and (23), as shown in Tables 17 and 18.

Table 17: Distance between the dominance evaluation value for the positive ideal solution (PIS)

	A	B	C	D	E	F	G	H
D_t^+	0.029	0.016	0.022	0.017	0.035	0.027	0.011	0.022

Table 18. Distance between the dominance evaluation value for the negative ideal solution (NIS)

	A	B	C	D	E	F	G	H
D_t^-	0.024	0.028	0.028	0.028	0.020	0.024	0.033	0.031

Finally, using (24), the priority value of each mill type is obtained (Table 19).

Table 19: Priority value of each selection

	A	B	C	D	E	F	G	H
B_t	0.445	0.641	0.564	0.627	0.361	0.469	0.742	0.585

From Table 19, the impact mill (G) has the highest priority value (0.742). Therefore, the impact mill is selected as the most suitable grinding machine for the dental gypsum grinding process.

3.2. Type Selection of Air Classifier

One of the major technical requirements in establishing the dental gypsum grinding–classification process is to ensure sufficient particle size distribution characteristics of the final powder product.

For grinding–classification processes requiring high classification accuracy, precision classifiers are generally used, particularly centrifugal air classifiers operating in a centrifugal force field.

In this study, four typical air classifier types are considered:

- O-Sepa type (a)
- ATP type (b)
- MS type (c)
- MSS type (d)

A suitable classifier is selected using the same fuzzy group decision-making methodology.

3.2.1. Setting the Main Criteria for Selecting the Type of Air Classifier

Ten technicians and experts serve as decision makers. The opinion data for selecting the main criteria among eleven possible criteria are listed in Table 20.

Table 20: Opinion data of the decision makers for selecting the main criteria of air classifier

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier	Manufacturing cost	Manufacturing Possibility	Easy of repair	suitability of operation	Life of classifier
1	0.8	0.8	0.8	0.8	0.7	0.9	0.4	0.3	0.4	0.4	0.4
2	0.9	0.8	0.9	0.7	0.6	0.8	0.3	0.3	0.3	0.4	0.5
3	0.8	0.7	0.9	0.8	0.7	0.8	0.4	0.3	0.4	0.3	0.4
4	0.7	0.7	0.7	0.7	0.7	0.8	0.5	0.3	0.3	0.4	0.4
5	0.9	0.7	0.9	0.8	0.7	0.8	0.3	0.3	0.4	0.5	0.5
6	0.7	0.6	0.7	0.6	0.6	0.7	0.4	0.3	0.3	0.4	0.4
7	0.9	0.7	0.8	0.8	0.7	0.9	0.5	0.2	0.4	0.5	0.5
8	0.8	0.6	0.8	0.7	0.7	0.9	0.2	0.2	0.3	0.3	0.4
9	0.8	0.7	0.8	0.8	0.8	0.9	0.3	0.3	0.3	0.4	0.5
10	0.9	0.8	0.8	0.8	0.7	0.9	0.2	0.3	0.3	0.4	0.4

Using the same procedure as in Section 3.1.1, the following six main criteria are identified:

1. Cut size
2. Classification efficiency
3. Classification accuracy
4. Capacity of classifier
5. Power consumption
6. Reliability of classifier

3.2.2. Determination of Criteria Weights Using FSFDMW

The importance weights of the six criteria are determined using the subjective opinion data of the ten decision makers (Table 21).

Table 21: Opinion data of 10 decision makers for determining the weight value of criteria in air classifier

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier
1	0.7	0.7	0.6	0.5	0.5	0.8
2	0.6	0.6	0.7	0.6	0.5	0.8
3	0.7	0.7	0.8	0.5	0.6	0.7
4	0.7	0.6	0.7	0.6	0.5	0.8
5	0.8	0.5	0.7	0.5	0.6	0.7
6	0.7	0.7	0.6	0.7	0.5	0.8
7	0.6	0.6	0.7	0.7	0.6	0.8
8	0.8	0.7	0.6	0.5	0.5	0.7
9	0.7	0.5	0.7	0.6	0.6	0.8
10	0.6	0.6	0.7	0.6	0.5	0.7

Using the corresponding equations (as applied in mill selection), the fuzzy score function values and normalized weight values are obtained (Table 22).

Table 22. Fuzzy score function values and the normalized weight values of the criteria

	Cut size	Classificatio efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier

S_i (H_6)	0.068995772	0.0620513	0.0680252	0.0579487	0.0539496	0.0760504
W_{xj}	0.17828	0.16021	0.17571	0.14988	0.13953	0.19639

3.2.3. Selection of Air Classifier Type Using FSFDMW–TOPSIS

The dominance evaluation opinion data for each criterion are provided in Tables 23–28.

Table 23. Opinion data of decision makers for estimating dominance to cut size (c_1)

	a	b	c	d
1	0.8	0.9	0.8	0.9
2	0.7	0.8	0.8	0.9
3	0.8	0.8	0.8	0.8
4	0.8	0.9	0.8	0.9
5	0.8	0.9	0.8	0.9
6	0.7	0.8	0.7	0.8
7	0.8	0.9	0.8	0.9
8	0.9	0.9	0.9	0.9
9	0.8	0.9	0.8	0.9
10	0.8	0.8	0.8	0.8

Table 24: Opinion data of decision makers for estimating dominance to classification efficiency (c_2)

	a	b	c	d
1	0.8	0.8	0.8	0.9
2	0.8	0.8	0.8	0.8
3	0.9	0.9	0.9	0.9
4	0.8	0.9	0.8	0.9
5	0.8	0.8	0.8	0.8
6	0.9	0.9	0.9	0.9
7	0.8	0.8	0.8	0.8
8	0.8	0.8	0.8	0.8
9	0.8	0.9	0.8	0.9
10	0.8	0.8	0.8	0.8

Table 25: Opinion data of decision makers for estimating dominance to classification accuracy (c_3)

	a	b	c	d
1	0.8	0.8	0.8	0.8
2	0.8	0.8	0.9	0.8
3	0.8	0.8	0.8	0.8
4	0.9	0.9	0.9	0.9
5	0.9	0.8	0.9	0.8
6	0.8	0.8	0.8	0.8
7	0.8	0.8	0.9	0.8
8	0.8	0.8	0.8	0.8
9	0.9	0.8	0.9	0.8
10	0.8	0.8	0.9	0.8

Table 26: Opinion data of decision makers for estimating dominance to capacity of classifier (c_4)

	a	b	c	d
1	0.7	0.6	0.8	0.7

2	0.8	0.7	0.8	0.7
3	0.7	0.6	0.8	0.6
4	0.7	0.6	0.7	0.6
5	0.7	0.7	0.8	0.7
6	0.8	0.8	0.8	0.8
7	0.7	0.6	0.8	0.6
8	0.8	0.7	0.8	0.7
9	0.7	0.6	0.7	0.6
10	0.7	0.7	0.8	0.7

Table 27: Opinion data of decision makers for estimating dominance to power consumption (c_5)

	a	b	c	d
1	0.5	0.6	0.8	0.6
2	0.6	0.7	0.7	0.5
3	0.7	0.7	0.9	0.7
4	0.6	0.8	0.8	0.7
5	0.7	0.7	0.8	0.6
6	0.7	0.67	0.9	0.7
7	0.6	0.7	0.9	0.7
8	0.7	0.8	0.9	0.7
9	0.7	0.7	0.8	0.7
10	0.6	0.7	0.8	0.7

Table 28: Opinion data of decision makers for estimating dominance to reliability of classifier (c_6)

	a	b	c	d
1	0.8	0.8	0.8	0.8
2	0.9	0.9	0.9	0.9
3	0.8	0.8	0.8	0.8
4	0.8	0.7	0.8	0.7
5	0.7	0.7	0.8	0.7
6	0.7	0.6	0.8	0.7
7	0.6	0.6	0.7	0.6
8	0.7	0.6	0.8	0.6
9	0.8	0.7	0.9	0.7
10	0.7	0.6	0.8	0.6

Using (14)–(18), the fuzzy score function value matrix is calculated (Table 29).

Table 29: Fuzzy score function value matrix with weights of decision makers

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier
a	0.2376	0.2485	0.2505	0.2575	0.2253	0.2525
b	0.2590	0.2514	0.2449	0.2315	0.2499	0.2338
c	0.2410	0.2459	0.2597	0.2758	0.2928	0.2761
d	0.2624	0.2542	0.2449	0.2352	0.2321	0.2377

The weighted decision-making matrix is obtained using (19) (Table 30).

Table 30: Decision making matrix with the fuzzy score function values and the weights of criteria

	Cut size	Classification efficiency	Classification accuracy	Capacity of classifier	Power consumption	Reliability of classifier
a	0.04242	0.03981	0.04402	0.03855	0.03142	0.04958
b	0.04623	0.04028	0.04305	0.03465	0.03484	0.04592
c	0.04302	0.03940	0.04564	0.04128	0.04083	0.05422
d	0.04684	0.04072	0.04305	0.03520	0.03237	0.04668

The PIS and NIS are determined using (20)–(23), and the corresponding distances are shown in Tables 31 and 32.

Table 31: Distance between the dominance evaluation value for the positive ideal solution (PIS)

	a	b	c	d
D_t^+	0.01185	0.01249	0.00404	0.01312

Table 32: Distance between the dominance evaluation value for the negative ideal solution (NIS)

	a	b	c	d
D_t^+	0.10126	0.10068	0.10861	0.10085

Using (24), the priority values are obtained (Table 33).

Table 33: Priority value of each selection

	a	b	c	d
B_t	0.89524	0.88966	0.96417	0.88490

From Table 33, the MS type air classifier (c) has the highest priority value (0.96417). Therefore, the MS type air classifier is selected as the most suitable classifier for the dental gypsum grinding–classification process (Fig. 1).

Fig 1. Dental gypsum grinding-classification process

4. CONCLUSION

In dry grinding–classification process design, the reliable selection of grinding machines and air classifiers is a critical issue due to the absence of statistical data and the presence of uncertainty in expert judgments. This study proposed a fuzzy group decision-making framework that integrates fuzzy equivalence clustering, fuzzy score function with decision makers' weights (FSFDMW), and the TOPSIS method to improve the rationality and accuracy of type selection under uncertain environments.

First, a method based on fuzzy equivalence clustering was developed to identify the main criteria influencing equipment selection. Then, an n-dimensional fuzzy environment was constructed to determine the weights of decision makers and criteria more realistically. Finally, a TOPSIS-based decision-making procedure using fuzzy score functions with decision maker weights was implemented to rank the available alternatives.

Application of the proposed method to the dental gypsum grinding–classification process demonstrated that five main criteria govern grinding mill selection, namely fineness of grinding product, reliability, content of impurities, capacity, and specific power consumption. Among eight candidate mills, the impact mill (G) achieved the highest priority value (0.742) and was identified as the most suitable grinding machine.

For air classifier selection, six main criteria were identified, including cut size, classification efficiency, classification accuracy, capacity, power consumption, and reliability. Among the four alternatives, the MS type air classifier (c) obtained the highest priority value (0.96417) and was selected as the optimal classifier.

The proposed FSFDMW–TOPSIS framework improves the reliability of group decision making by incorporating realistic weighting of decision makers while maintaining computational simplicity. Therefore, it provides a practical and systematic tool for equipment type selection in grinding–classification process design under uncertainty.

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